

Practicalities of operational risk capital modelling

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SFMW Workshop
10 September 2013

Topics

1. Overview of operational risk capital
2. Modelling components
3. Some modelling challenges
4. Some business challenges
5. Possible future directions
6. Useful reading

1. Overview of operational risk capital

- What is operational risk capital?
- Regulatory capital
- Economic capital
- Recent trends

What is operational risk capital?

- Operational risk is defined in Basel II/III as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This includes legal risk, but excludes strategic and reputational risk.
- Discussion of motivating examples – money laundering, London whale
- Operational risk capital in this talk is the capital that banks hold for operational risk. It is usually driven by the risk of single large events.

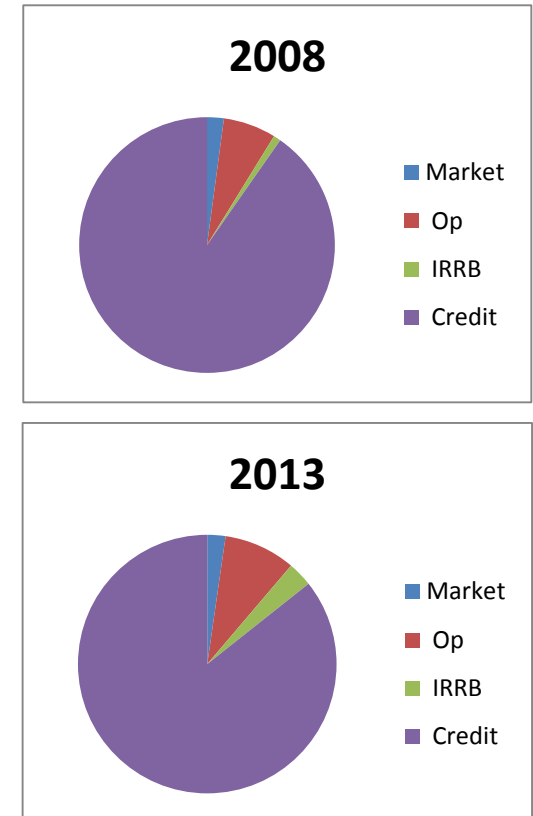
Regulatory capital

- Under Basel II/III, operational risk capital is measured for regulatory capital purposes either under the Basic Indicator Approach, the Standardised Approach or the Advanced Approaches
- The Basic Indicator Approach uses a proportion (15%) of gross income, averaged over a three year period
- The Standardised Approach uses a similar idea, but is more granular, using different proportions of gross income for different businesses ranging from 12% to 18%
- There is also an Alternative Standardised Approach, where for two businesses (retail and commercial) the income number is replaced by a fixed (3.5%) proportion of assets
- The Advanced approaches involve the use of own models, as accredited by the regulator

Regulatory capital

- In Australia the relevant standards are APS 114 (Standardised Approach) and APS 115 (Advanced Approaches or AMA)
- Risk weighted assets are calculated by multiplying the capital number by 12.5, and this number feeds into the relevant capital ratios
- Operational risk capital modelling is a rapidly growing area of quantitative analysis
- The four majors' risk weighted assets corresponding to operational risk have risen from just under 7% of total RWA in 2008 to about 9% in 2013
- These are part of the Pillar 1 capital requirements

RWAs Four Majors



Source: Pillar 3 reports

Economic capital

- Economic capital is the capital that banks calculate internally to account for operational risk – comes under Pillar 2
- Economic capital usually has a wider range of internal uses than regulatory capital e.g.
 - Solvency determination
 - Earnings volatility assessment
 - Risk appetite quantification
 - Limit setting mechanisms
 - Risk adjusted performance measurement for the bank, business units, products and even individuals (RAROC type measures)
 - Portfolio optimisation (maximising RAROC)
- Some of the key modelling differences can be:
 - Different confidence interval for economic capital
 - Different diversification benefits for economic capital
 - Some entities (e.g. Level 3) included for economic capital but not regulatory

Recent trends

- There is increasing focus on the management and measurement of operational risk in banks globally, relating principally to:
 - Significant attention from regulators
 - A background of some large operational risk losses and the problems of the GFC (e.g. CROPS)
 - The use of capital models in the business to gain insight into the appetite for and management of operational risk. The risk sensitivity of the models to changes in risk profile and how this informs business decisions is key.
- Models are becoming more granular in response to this, moving from “top of house” in the first generation to lower business lines (BL) and individual event types (ET) in the second generation e.g. 8 business lines and 7 event types in Basel II (56 categories)
- Banks are exploring methods of model design and capital allocation to make the numbers more meaningful to the business
- There is also work on a range of new technical developments

2. Modelling components

- LDA model
- Frequency
- Severity
- Aggregation
- Dependency
- Allocation
- Data

LDA model

- The most common approach to modelling operational risk is the Loss Distribution Approach (LDA), where Frequency and Severity distributions are combined to produce a Loss Distribution
- The LDA approach is a method that is quite familiar in actuarial circles, as are many of the distributions involved
- The problem of combining the two distributions can be solved by Monte Carlo simulation or numerical methods in some cases
- Capital is determined as a confidence interval on the loss distribution (99.90% under Basel II and APS 115)



Frequency component

- Frequency is usually modelled by a Poisson distribution or a negative binomial distribution, but sometimes just by an empirical distribution
- The negative binomial is sometimes preferred to the Poisson when the variance exceeds the mean (it can be seen as a gamma Poisson mixture, where the Poisson parameter is gamma distributed)
- In actual fact, the nature of the distribution for frequency turns out to be an issue of less importance than the choice for severity, as we shall see later

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \mu = \lambda, \sigma^2 = \lambda$$

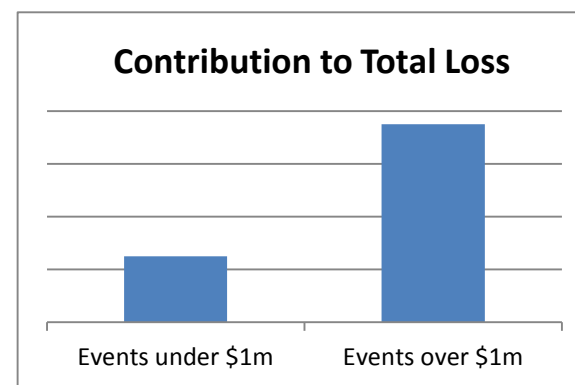
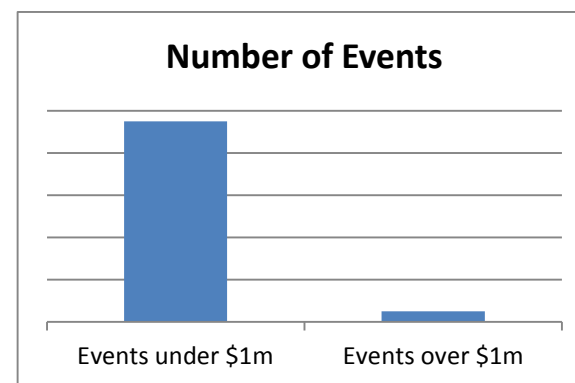
or

$$f(x) = {}^{x+r-1}C_{r-1} (1-p)^x p^r, \mu = \frac{r(1-p)}{p}, \sigma^2 = \frac{\mu}{p}$$

Severity component

- Although much rarer than small losses, large losses occur frequently enough and are large enough to dominate the severity distributions, which are typically heavy tailed
- This brings the theory into the interesting area of Extreme Value Theory (EVT)
- The essential contrast with credit modelling is that large loss years tend to be caused by individual large losses, whereas in credit they tend to occur when correlation is high and a lot of loans default together
- Severity distributions tend to be modelled in two parts (using shifting or truncation) because of the different characteristics of the body and the tail

Typical behaviour



Subexponential distributions

A distribution $F(x)$, $x \in (0, \infty)$, is subexponential if

$$\lim_{x \rightarrow \infty} \frac{\overline{F^{(n)*}}(x)}{\overline{F}(x)} = n, n \geq 2$$

The term subexponential comes from the fact that for these distributions

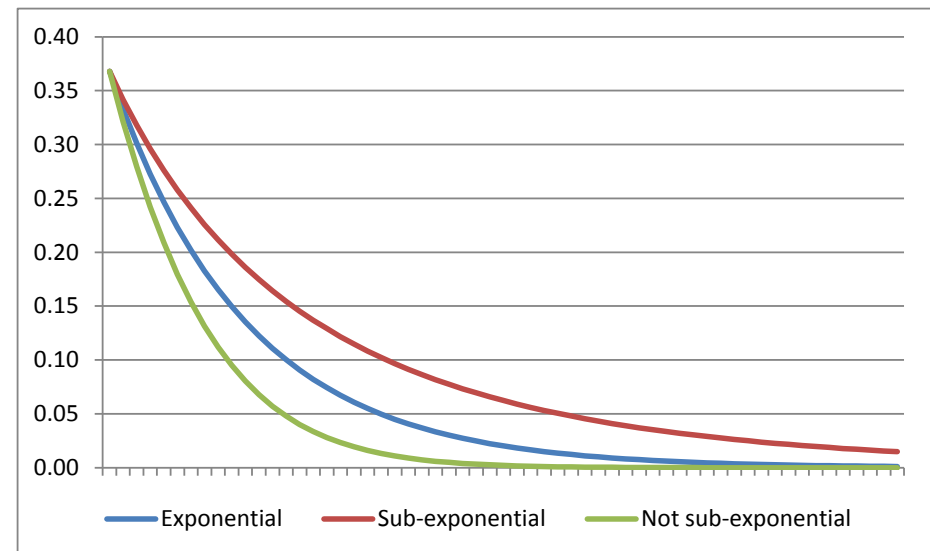
for all $\varepsilon > 0$, $e^{\varepsilon x} \overline{F}(x) \rightarrow \infty$

Good examples :

Weibull : $\overline{F}(x) = \exp(-(x/\beta)^\gamma)$, $\gamma < 1$

Pareto : $\overline{F}(x) = (1 + x/\beta)^{-\alpha}$

Lognormal : $\overline{F}(x) = 1 - N\left[\frac{\ln(x) - \mu}{\sigma}\right]$



Aggregation

- The standard aggregation technique is Monte Carlo
- Useful numerical techniques are Fast Fourier Transform and Panjer Recursion
- A very useful analytical approximation is the Single Loss Approximation (see next slide)

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
No. losses	9	3	5	2	6	10	9	2	1	7
Losses	0	2	0	17	2	1	0	53	19	9
	0	30	6	1	0	0	0	0		0
	1105	1	0		0	0	2			3
	0		1		0	3	13			0
	38		200		2	0	1			0
	12				3	1	0			0
	0					79	0			5
	23					1	1			
	3					0	0			
						4				
Sum	1181	33	208	18	7	86	17	53	19	17
Max/Sum	93%	92%	96%	93%	40%	93%	75%	99%	100%	51%

Sums of subexponential distributions

If $X_1, \dots, X_n$ are independent subexponentials with common CDF $F(x)$

$$\lim_{x \rightarrow \infty} \frac{\Pr[X_1 + \dots + X_n > x]}{\Pr[\max(X_1, \dots, X_n) > x]} = 1$$

If N is random (with $p_n = \Pr(N = n)$) satisfying $\sum_0^\infty (1 + \varepsilon)^n p_n < \infty$ for some $\varepsilon > 0$

(e.g. Poisson) then $\bar{G}(x) \sim E(N) \bar{F}(x)$, where G is the CDF of the sum $X_1 + \dots + X_N$

This gives rise to the Single Loss Approximation (Böcker and Klüppelberg (2005)):

$$G^{-1}(p) \sim F^{-1}\left(1 - \frac{1-p}{E(N)}\right), \text{ or as amended by Degen (2010):}$$

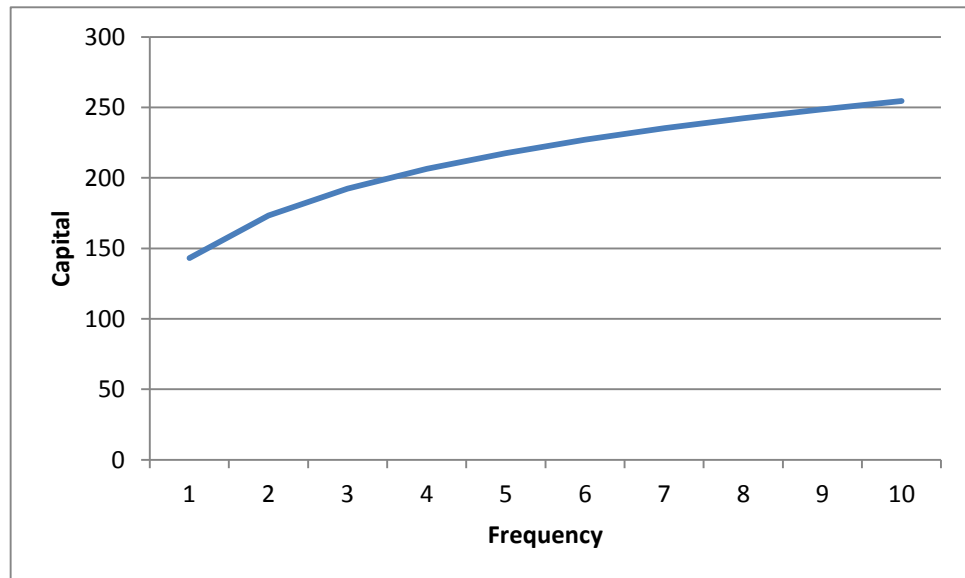
$$G^{-1}(p) \sim F^{-1}\left(1 - \frac{1-p}{E(N)}\right) + \mu E(N)$$

Example of Single Loss Approximation

$E(N) = 5$, F is Weibull(3, 0.5), so $\bar{F}(x) = \exp(-(x/3)^{0.5})$, $p = 0.999$

$$G^{-1}(p) = F^{-1}\left(1 - \frac{1-p}{E(N)}\right) = 3\left[-\ln\left(1 - \left(1 - \frac{1-0.999}{5}\right)\right)\right]^{1/0.5} = 218$$

Freq	Adj CI	Capital
1	0.999000	143
2	0.999500	173
3	0.999667	192
4	0.999750	206
5	0.999800	218
6	0.999833	227
7	0.999857	235
8	0.999875	242
9	0.999889	249
10	0.999900	254



Note: In practice, need to be careful with thresholds

Aggregation gets more complicated

	Internal Fraud	External Fraud	Employment practices and workplace safety	Clients, products and business practices	Damage to physical assets	Business disruption	Execution, delivery and process management
Corporate Finance							
Trading and Sales							
Retail Banking							
Commercial Banking							
Payment and Settlement							
Agency Services							
Asset Management							
Retail Brokerage							

Sums of compound Poisson

*Suppose that compound sums $S_{N_i} \sim \text{CPoi}(\lambda_i, G_i), i = 1, \dots, d$,
and that these random variables are independent,
then $S_N = \sum_1^d S_{N_i} \sim \text{CPoi}(\lambda, G)$, where $\lambda = \sum_1^d \lambda_i$ and $G = \sum_1^d (\lambda_i / \lambda) G_i$*

*So the new intensity is just the sum of the old ones and the new severity
is a discrete mixture of the loss densities with weights λ_i / λ*

*Reference : Quantitative Risk Management. McNeil, Frey, Embrechts
(Princeton 2005)*

Dependency

- Various types of dependency across categories are at play here:
 - Frequency
 - Severity
 - Loss
 - Scenarios
 - BUT not all of these are possible under the LDA
- Copulas are usually used to bring marginal distributions together
- Because of the dominance of large individual losses in severity modelling with subexponential distributions, modelling dependency mostly comes down to a consideration of tail dependency. This is again a difficult area because of the paucity of data.
- In any event, it is not clear how much difference dependency makes at high percentiles, given that losses are driven by individual large events:

$$\bar{G}(x) \sim E(N) \bar{F}(x)$$

Data inputs

Term	Meaning	Characteristics
ILD	Internal loss data	Not generally available in large quantities. Can lack extreme events. Used for frequency, severity.
ELD	External loss data	Several sources, notably ORX (consortium of around 60 banks). Used for frequency, severity.
Scenarios	Scenarios	Developed internally to counter data problem on large losses but also for risk management. Principally used for severity. More prevalent in Europe than the US (Atlantic divide).
BEICF	Business environment and internal control factors	Largely indirect input to model (say via scenarios). Quantitative measures still not widely available.

3. Some modelling challenges

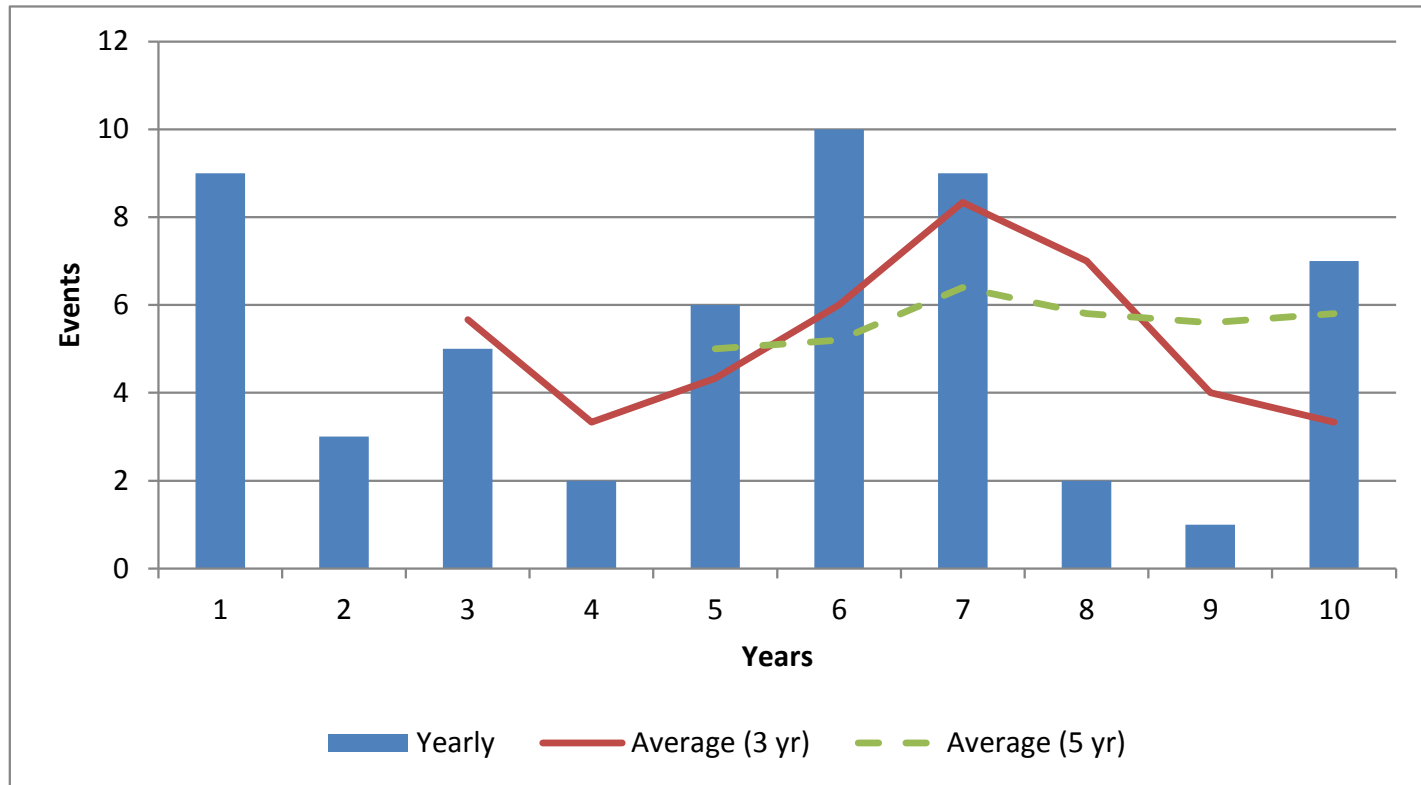
- Frequency
- Severity
- Dependency (already discussed)
- Allocation

Frequency challenges

- The initial approach to estimating frequency is usually to use historical loss events at the bank
- The main challenges here are:
 - Frequency is dominated by smaller events
 - Is the frequency process homogeneous?
 - What about if there is not enough data (presumably a good situation) especially at the more granular level of Business Line/Event Type?
 - What is the best form of weighting/prediction to use on historic data?
- There are another set of challenges in how to incorporate a “forward looking” element to the frequency (e.g. involvement of BEICF)

Frequency challenges

Frequencies generated from random Poisson



Severity challenges

- Because of the paucity of data relating to large losses, the severity distributions are very difficult to parameterise
- As mentioned earlier, most fitting is done above a threshold, using truncation or shifting
- Standard fitting techniques can be very volatile under data changes and are susceptible to outliers
- For example, if samples of 100 points come from a Lognormal (14,2) distribution, then the standard error in estimating the mean μ is 0.20. The capital impact of this standard error is about 20%, although this would normally be for a category, not the top of house number
- Maximum Likelihood Estimation is not always the best method and other techniques may need to be explored, such as quantile distance fitting or minimising GOF statistics
- Robust Statistics has a role to play here

Allocation challenges

- There are standard techniques for allocating from the total loss distribution down to sub-distributions e.g. risk contribution

$$\sigma_P^2 = \sum \sum \rho_{ij} \sigma_i \sigma_j, \quad RC_i = (\sigma_i \sum \rho_{ij} \sigma_j) / \sigma_P$$

- Related concepts are Euler allocation, expected shortfall contribution (see for example the work of Tasche)
- These are not always conducive to the business use of the model e.g. low diversification benefit to the largest contributors to capital and high diversification benefit to the low contributors
- Need to work closely with the business on this

Example

$$\sigma_1 = 1, \sigma_2 = 10, \rho_{12} = 0 \Rightarrow \sigma_P = 10.05, RC_1 = 0.10, RC_2 = 9.95$$

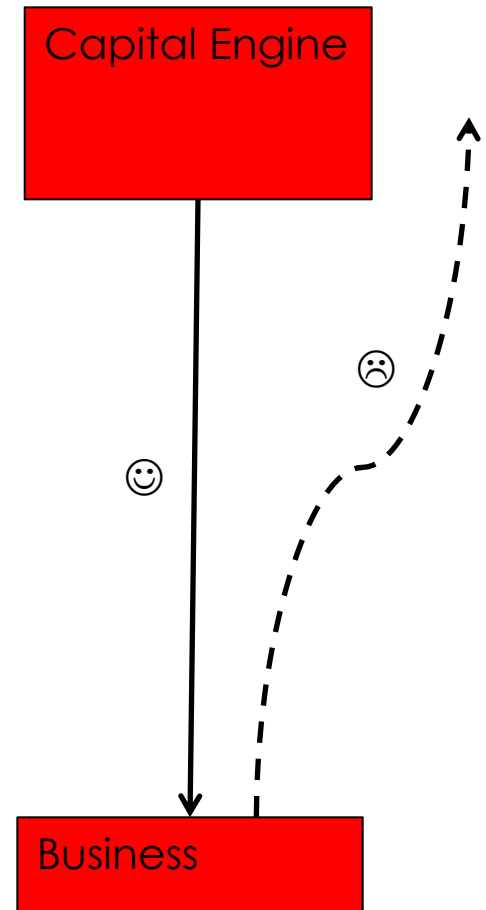
Diversification benefits of 90% and less than 1% respectively

4. Some business challenges

- Allocation
- Model maturity
- Scenarios
- Balancing sensitivity and volatility

Allocation

- Businesses don't always see a connection between their actions and capital changes, as the “top down” approach to operational risk modelling may make an understanding of the levers that can move capital more difficult
- It can even be that in some models the levers that are used for allocation (e.g KPIs) are not actually connected to the modelling engine. This can mean that improved or deteriorating conditions result in a redistribution of capital, not an overall decrease or increase.
- This is in contrast to say credit, where the PD, LGD scheme is generally well connected to capital



Model development

- Models are complex and are still developing. There is no single accepted paradigm.
- Models were at an earlier level of maturity than in credit when Basel II accreditation happened and businesses have been subjected to a lot of experimentation!
- There is another strong contrast with credit that makes a real difference in understanding risk profile in that there is no natural definition of “exposure” to a risk category, even when one understands the risk characteristics of different categories
- In credit, this is covered by the concept of EAD
- The ambiguity in dealing with both regulatory and economic capital can also be problematic, as in other risk classes

Use of scenarios

- The scenario process is special in the crucial role that it plays in operational risk capital modelling, because of the perceived lack of forward looking severe loss data
- However, the eliciting of accurate outputs is very challenging
 - Which outputs are being sought?
 - What is the likelihood of the outputs?
 - How should the outputs be used in influencing the parameterisation of the severity distribution?
- There are divided views on allowing a subjective judgment to feed into the modelling process and the appropriate degree of “blending” with ELD

Weibull example where $p = F(x) = 1 - \exp(-(x/\beta)^\gamma)$

If the scenario delivers x at p and we know β , we can deduce γ or

if we know γ we can deduce β

Balancing volatility and sensitivity

- Because of the sensitivity of parameter choices to variations in data, particularly in relation to severity, stability of capital outputs is a problem if “best fits” are applied to the distributions involved and refreshed very frequently
- Some degree of classification and stability is needed in the parameter fitting
- Against this, the models need to be sufficiently sensitive to changes in risk profile, but this always has to be explainable to the business:
 - Businesses will find it easy to understand if their capital changes because of some direct change to their business (data or scenarios)
 - They will find it less easy but not impossible if the change is in the external environment or happens to some other business
 - They will find it impossible if the reason is “that’s just what the model says”

5. Future directions

- A new generation of models is being built
- Increasing regulator focus (e.g. SIGOR work)
- Increasing engagement from bank senior management
- Industry forums are facilitating development
- Better data is evolving internally to banks and with industry initiatives e.g. ORX
- Discussions around more simplicity rather than more complexity. Further convergence of AMA models is possible (also AMA and Standardised).
- New research
 - Severity
 - Dependency
 - Allocation
 - More focus on drivers of operational risk e.g. factor based models
 - More granularity and attempts to build “bottom up” models e.g. exposure based

6. Useful reading

1. Journal of Operational Risk. Risk Journals, Quarterly
2. Modelling Extremal Events. Embrechts, Klüppelberg, Mikosch (Springer, 1997)
3. Modelling Operational Risk Using Bayesian Inference. Pavel Schevchenko (Springer, 2011)
4. Operational Risk – Supervisory Guidelines for the Advanced Measurement Approaches. Basel Committee on Banking Supervision, June 2011
5. ORX website, www.orx.org
6. Prudential Standards APS 114 and APS 115. APRA (latest January 2013)